Table 1 Thermal conductivities of gaseous unsymmetrical dimethylhydrazine

	k, (cal-cm^{-1}-sec-$^{-1}$-°C^{-1}) × 10^7	T,°C
Experimental	272	5.7
	274	9.2
	293	29.5
	301	33.1
Calculated	266	0
(on basis of	276	10
linear variation	286	20
with temperature)	296	30
	306	40
	316	50

filament was always less than the filament temperature. Therefore, the first values of α calculated were used to estimate the true gas temperatures at the filament from Eq. (7):

$$\alpha = \Delta t(\text{true})/\Delta t(\text{experimental}) \tag{7}$$

Thermal conductivities then were adjusted downward to conform to the average temperatures $t_{g(1)} + t_2/2$, where $t_{g(1)}$ denotes the first calculated temperature of the gas at the filament. This procedure was repeated until calculated values of α converged.

Calibration curves of α vs molecular weight of standard gas were plotted for the four experimental environments used: 1) minimum voltage, 0°C bath; 2) maximum voltage, 0°C bath; 3) minimum voltage, room temperature bath; and 4) maximum voltage, room temperature bath. Thermal conductivities for unsymmetrical dimethylhydrazine were calculated from Eq. (3) by selecting the values of α appropriate to its molecular weight of 60.08 and each experimental environment. (The value of α varied from 0.655 to 1.025). The mean temperature applicable to each thermal conductivity was given by

$$\langle t\rangle = t_2 + [\alpha(t_1 - t_2)/2] \tag{8}$$

Results

Thermal conductivities of gaseous unsymmetrical dimethylhydrazine are presented in Table 1. On the basis of linear variation with temperature, the change in thermal conductivity with respect to change in temperature, $\Delta k/\Delta T$, is very close to 10^{-7} cal/cm-sec-°C^2.

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Second Approximation to the Solution of the Rendezvous Equations

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The equations of motion of a satellite relative to a coordinate system whose origin moves in a circular satellite orbit are solved to second order by the method of successive approximations. The closed-form solution thus obtained provides a second-order correction to the standard linear solution and, on the basis of sample cases representative of rendezvous conditions, appears to decrease greatly the numerical errors resulting from the use of the linear solution alone.

Received January 14, 1963.

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The familiar linear approximation to the solution of the equations of relative motion of a satellite moving in a circular orbit and a nearby satellite has been used extensively in studies of rendezvous operations, e.g., Refs. 1 and 2. It has been recognized, however, that the numerical accuracy of the linear solution is not adequate for many cases of interest.[3] Errors that increase with flight time result from neglecting second- and higher-order terms in various components of relative distance (range) and relative velocity (range rate) normalized with respect to orbital radius and orbital velocity, respectively, depending upon the particular coordinate system used.[4]

Since the range and range rate are small in most cases of current interest, it is felt intuitively that the accuracy of the linear solution can be improved considerably by adding a second-order correction. The second-order correction in rectangular coordinates is obtained herein by the method of successive approximations, i.e., second-order gravitational terms are retained in the differential equations of relative motion and approximated by the results of the linear solution. These terms thereby become time-dependent forcing functions in a set of linear differential equations for the second-order correction; the solution is obtained in closed form as a function of time and the initial values of the components of range and range rate.

A rectangular coordinate system is chosen such that its origin moves in a circular orbit of radius r_0 with the angular speed ω corresponding to satellite speed at r_0; the x-y plane is in the plane of this orbit and the z direction normal to it. The equations of relative motion of a satellite are, in dimensionless form,

$$\ddot{x} - 2\dot{y} + x[(1/r^3) - 1] = 0 \tag{1}$$

$$\ddot{y} + 2\dot{x} + (1 + y)[(1/r^3) - 1] = 0 \tag{2}$$

$$\ddot{z} + (z/r^3) = 0 \tag{3}$$

$$r = (1 + 2y + x^2 + y^2 + z^2)^{1/2} \tag{4}$$

where x, y, z, and r are normalized with respect to the orbital radius r_0; $\dot{x}$, $\dot{y}$, and $\dot{z}$ are normalized with respect to orbital velocity at r_0, and the dot notation indicates differentiation with respect to the dimensionless time $\tau = \omega t$. The term r^{-3} is expanded in a binomial series and terms of third order and higher in Eqs. (1–3) are dropped, giving equations that are correct to second order

$$\ddot{x} - 2\dot{y} - 3xy = 0 \tag{5}$$

$$\ddot{y} + 2\dot{x} - 3y + 3y^2 - \tfrac{3}{2}(x^2 + z^2) = 0 \tag{6}$$

$$\ddot{z} + z - 3yz = 0 \tag{7}$$

Now let $x = x_1 + x_2$, $y = y_1 + y_2$, and $z = z_1 + z_2$ where subscript 1 denotes the first-order solution and subscript 2 denotes a second-order correction. It is assumed that if terms in the first-order solution are of order of magnitude δ, then terms in the second-order correction are of order δ^2. Accordingly, the first-order equations are

$$\ddot{x}_1 - 2\dot{y}_1 = 0 \tag{8}$$

$$\ddot{y}_1 + 2\dot{x}_1 - 3y_1 = 0 \tag{9}$$

$$\ddot{z}_1 + z_1 = 0 \tag{10}$$

and the equations for the second-order corrections, consistent with the order-of-magnitude assumptions, are

$$\ddot{x}_2 - 2\dot{y}_2 = 3x_1y_1 \tag{11}$$

$$\ddot{y}_2 + 2\dot{x}_2 - 3y_2 = -3[y_1^2 - \tfrac{1}{2}(x_1^2 + z_1^2)] \qquad (12)$$

$$\ddot{z}_2 + z_2 = 3z_1y_1 \qquad (13)$$

The latter equations also have been derived in Ref. 4. The well-known solution of the first-order equations (8–10) is

$$x_1 = (4\dot{x}_0 - 6y_0)\sin\tau - 2\dot{y}_0\cos\tau + (6y_0 - 3\dot{x}_0)\tau + x_0 + 2\dot{y}_0 \qquad (14)$$

$$y_1 = \dot{y}_0\sin\tau + (2\dot{x}_0 - 3y_0)\cos\tau + (4y_0 - 2\dot{x}_0) \qquad (15)$$

$$z_1 = \dot{z}_0\sin\tau + z_0\cos\tau \qquad (16)$$

These results then are substituted into the right-hand side of Eqs. (11–13), which can then be solved straightforwardly. Since the details are tedious, only the results are given here

$$x_2 = \alpha_0 + \alpha_1\tau + \alpha_2\sin\tau + \alpha_3\cos\tau + \alpha_4\sin2\tau + \alpha_5\cos2\tau + \alpha_6\tau\sin\tau + \alpha_7\tau\cos\tau \qquad (17)$$

$$y_2 = \beta_0 + \beta_1\tau + \beta_2\tau^2 + \beta_3\sin\tau + \beta_4\cos\tau + \beta_5\sin2\tau + \beta_6\cos2\tau + \beta_7\tau\sin\tau + \beta_8\tau\cos\tau \qquad (18)$$

$$z_2 = \gamma_0 + \gamma_1\sin\tau + \gamma_2\cos\tau + \gamma_3\sin2\tau + \gamma_4\cos2\tau + \gamma_5\tau\sin\tau + \gamma_6\tau\cos\tau \qquad (19)$$

The coefficients α_n, β_n, γ_n are second-degree functions of the initial values x_0, y_0, z_0, $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$ and are given in detail in the List of Coefficients.

It is important that the y and z motions, which are simple harmonic in the linear approximation, contain mixed terms (i.e., of the form $\tau\sin\tau$) in the second-order correction and that the y motion contains secular terms, τ and τ^2, as well.

Numerical Results

The second-order results are compared in two different cases with calculations based on the first-order theory and with numerical integration of the complete differential equations. The author is grateful to Henry Pearson of the Flight Mechanics Branch of the NASA Langley Research Center for making available the numerical integration data.

Two sample cases were selected arbitrarily. For simplicity, both are cases of coplanar motion, i.e., $z_0 = \dot{z}_0 = 0$, and the calculations are initiated with the satellite at the origin of the rotating coordinate system, i.e., $x_0 = y_0 = 0$. This, of course, actually would be the situation at the end of an actual rendezvous maneuver, prior to an impulsive velocity correction, rather than the beginning. In both cases the origin of the rotating coordinate system is moving in a 300-statute-mile-alt earth orbit.

In the first case, calculations are carried out in positive time so that the simulated motion is that of the subsequent drifting away from the origin if a velocity correction is not made. The initial relative velocity, ΔV, is 200 fps and is at angle θ of 45°, where $\theta = \tan^{-1}\dot{y}/\dot{x}$. Results are shown in Fig. 1 where the relative vertical displacement y is plotted vs time. It is seen that the linear theory begins to diverge from the numerical integration results after half a revolution. The second-order correction, however, brings the analytical results into almost exact agreement with the numerical integration until the end of the second revolution at which time the apparent error is reduced from about 88 miles to about 5 miles. In the second case, the initial relative velocity is 800 fps at an angle θ of 70° and the calculations are carried out in negative time. This is one of a series of rendezvous trajectories studied in Ref. 3 in which the trajectory is integrated backward to booster burnout conditions at 60-statute-mile-alt, occurring in this case at $\tau = -120°$. Here again, as shown in Fig. 2, the second-order correction brings the linear theory into excellent agreement with the numerical integration reducing the error at the burnout point from about 23 miles to about 1 mile.

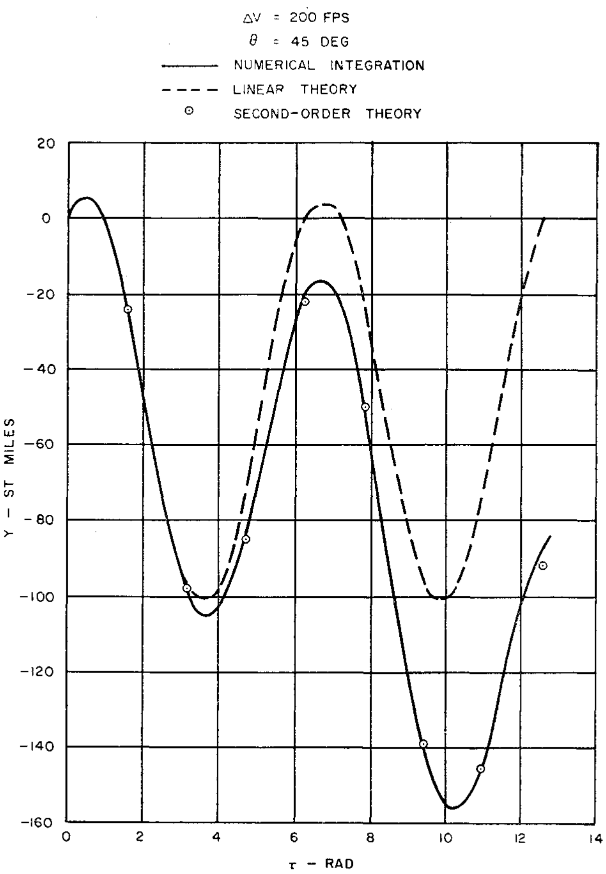

Fig. 1 Vertical component of relative motion

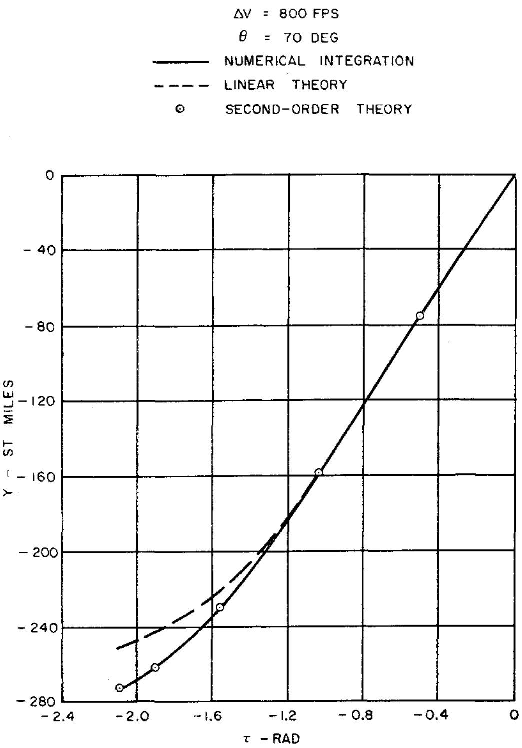

Fig. 2 Vertical component of relative motion

List of Coefficients α, β, γ

$$\alpha_0 = 3[x_0y_0 - \dot{x}_0\dot{y}_0 + \tfrac{5}{2}y_0\dot{y}_0 + \tfrac{1}{2}z_0\dot{z}_0]$$

$$\alpha_1 = 3[x_0^2 + \tfrac{11}{2}y_0^2 + \tfrac{1}{2}z_0^2 + 2\dot{x}_0^2 + \tfrac{1}{2}\dot{y}_0^2 + \tfrac{1}{2}\dot{z}_0^2]$$

$$\alpha_2 = 36\dot{x}_0y_0 - 30y_0^2 - 10\dot{x}_0^2 - 3x_0\dot{y}_0 - 2\dot{y}_0^2 - 3x_0^2 - z_0^2 - 2\dot{z}_0^2$$

$$\alpha_3 = -3x_0y_0 + 2\dot{x}_0\dot{y}_0 - 6y_0\dot{y}_0 - 2z_0\dot{z}_0$$

$$\alpha_4 = \tfrac{1}{4}\dot{y}_0^2 - \dot{x}_0^2 + 3\dot{x}_0y_0 - \tfrac{9}{4}y_0^2 + \tfrac{1}{4}\dot{z}_0^2 - \tfrac{1}{4}z_0^2$$

$$\alpha_5 = -\tfrac{1}{2}\dot{y}_0(3y_0 - 2\dot{x}_0) + \tfrac{1}{2}z_0\dot{z}_0$$

$$\alpha_6 = -3\dot{y}_0(2y_0 - \dot{x}_0)$$

$$\alpha_7 = -3(2y_0 - \dot{x}_0)(2\dot{x}_0 - 3y_0)$$

$$\beta_0 = 3[(x_0^2/2) + \dot{x}_0^2 + \tfrac{7}{2}y_0^2 - \tfrac{1}{2}\dot{y}_0^2 - 4\dot{x}_0y_0 + \tfrac{1}{4}(z_0^2 + \dot{z}_0^2)]$$

$$\beta_1 = -3(x_0 + 2\dot{y}_0)(2y_0 - \dot{x}_0)$$

$$\beta_2 = -\tfrac{9}{2}(2y_0 - \dot{x}_0)^2$$

$$\beta_3 = 12y_0\,\dot{y}_0 + 6x_0y_0 - 7\dot{x}_0\dot{y}_0 - 3x_0\dot{x}_0 + z_0\dot{z}_0$$

$$\beta_4 = -\tfrac{3}{2}x_0^2 - 5\dot{x}_0^2 - 15y_0^2 + 2\dot{y}_0^2 + 18\dot{x}_0y_0 - \dot{z}_0^2 - \tfrac{1}{2}z_0^2$$

$$\beta_5 = \dot{y}_0(2\dot{x}_0 - 3y_0) - (z_0\dot{z}_0/2)$$

$$\beta_6 = \tfrac{9}{2}y_0^2 + 2\dot{x}_0^2 - \tfrac{1}{2}\dot{y}_0^2 + \tfrac{1}{4}\dot{z}_0^2 - \tfrac{1}{4}z_0^2 - 6\dot{x}_0y_0$$

$$\beta_7 = -3(2\dot{x}_0 - 3y_0)(2y_0 - \dot{x}_0)$$

$$\beta_8 = 3\dot{y}_0(2y_0 - \dot{x}_0)$$

$$\gamma_0 = \tfrac{3}{2}[\dot{y}_0\,\dot{z}_0 + z_0(2\dot{x}_0 - 3y_0)]$$

$$\gamma_1 = 3y_0\dot{z}_0 - \dot{x}_0\dot{z}_0 + z_0\dot{y}_0$$

$$\gamma_2 = -\tfrac{3}{2}\dot{y}_0\dot{z}_0 - 2z_0\dot{x}_0 + 3y_0z_0 - \tfrac{1}{2}y_0\,\dot{z}_0$$

$$\gamma_3 = -\tfrac{1}{2}[\dot{z}_0(2\dot{x}_0 - 3y_0) + z_0\dot{y}_0]$$

$$\gamma_4 = -\tfrac{1}{2}[z_0(2\dot{x}_0 - 3y_0) - \dot{z}_0\dot{y}_0]$$

$$\gamma_5 = 3z_0\,(2y_0 - \dot{x}_0)$$

$$\gamma_6 = -3\dot{z}_0(2y_0 - \dot{x}_0)$$

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Determination of Hypersonic Flow Fields by the Method of Characteristics

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The conservation of mass, momentum, and energy in a hypersonic method-of-characteristics solution is examined and large errors are discovered. The source of the error is traced to an assumption used in the normal method of characteristics computer program and not in the theory itself. A new method of determining the local entropy in a rotational field is discussed and shown to reduce the conservation errors drastically.

Received April 3, 1963.

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In Ref. 1, a check was made of the continuity of mass, momentum, and energy in a hypersonic flow field calculated by the method of characteristics. On the basis of the errors found in this check, the authors concluded that certain errors were inevitable, and that corrections to the computed results must be made.

A similar check has been carried out for one problem of a previously published set of real gas method of characteristics solutions.[2] In contrast to Ref. 1 where the continuity checks were carried out in a constant X plane, the mass, momentum, and energy conservation equations were evaluated along right running characteristics from the shock to the body.

The body used in the problem under discussion is shown in Fig. 1. The nose sphere has a radius of 0.25, and is followed by a tangent frustum, which in turn is faired smoothly by a radius segment into a cylindrical afterbody of radius 1.0. The freestream Mach number used was 35 and an altitude of 300,000 ft. The mass-ratio results given in Fig. 1 typify all the results for momentum and energy also. Two curves are given: one showing the results from a conventional method of characteristics method, and one showing the results using a new method proposed below. It is evident that large errors in continuity can and do occur when using a conventional method.

In order to discover the reason for the large conservation errors, consider the method of calculating a normal field point in a rotational method of characteristics procedure. It is assumed that the independent variables are P and δ, the local pressure and flow direction, so that entropy does not occur explicitly in the compatibility relations. Figure 2 is a sketch of a field showing two known points, A and B, which are to be used in determining a third point, C. By using the compatibility relations in finite difference form, the data given at points A and B determine the pressure and flow direction at C. If $\tan\delta$ is assumed to vary linearly between A and B, a quadratic expression can be derived for determining point D, the intersection of the streamline passing through C with the line joining A and B. Since entropy is a constant along streamlines in equilibrium flows, a linear interpolation between A and B for the entropy at D then determines the entropy at C.

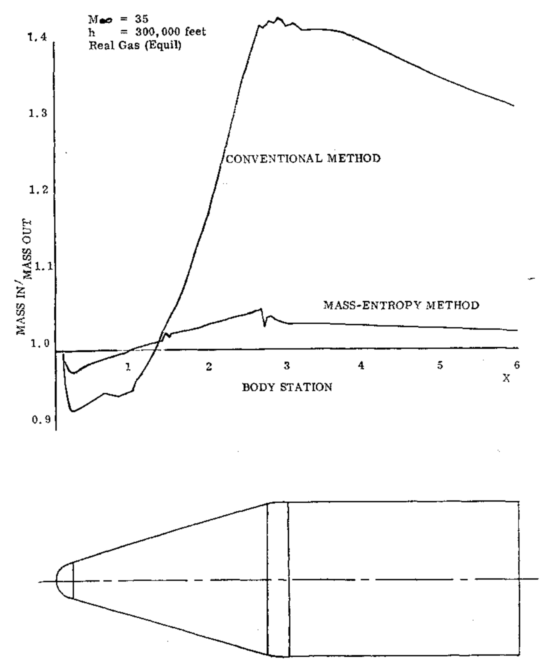

Fig. 1 Continuity check